Is There Too Much Benchmarking of Asset Managers?

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September 18, 2019

Abstract

We ask why benchmarks are so pervasive in asset management, and what the general-equilibrium effects of using them are. In our model, asset managers' portfolios are unobservable and the managers incur some private costs in running them. The managers are paid based on their performance. Conditioning asset managers' compensation on performance of a benchmark portfolio partially protects them from risk, and thus gives them incentives to generate higher returns. In general equilibrium, the use of such incentive contracts creates an externality through their effect on asset prices. Benchmarking inflates asset prices and gives rise to crowded trades, thereby reducing the effectiveness of incentive contracts for others. We show that privately-optimal contracts chosen by fund investors diverge from socially-optimal ones. A social planner, recognizing the crowding, opts for less benchmarking and less incentive provision. Privately-optimal contracts end up forcing asset managers to invest too much at too high a cost, and the planner corrects this. The planner's choice of benchmark portfolio weights also differs from the privately-optimal one.

JEL Codes: D82, D86, G11, G12, G23

Keywords: Asset Management, Benchmark, Index, Contracts, Compensation, Welfare, Externality

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We have benefited from comments and suggestions by seminar and conference participants at London Business School, CERGE-EI Macro Workshop, 2019 SAET Conference, as well as discussions with Hector Chade and Dimitri Vayanos. This research has been supported by a grant from the Alfred P. Sloan Foundation to the Macro Financial Modeling project at the University of Chicago. The views expressed here are ours only and not necessarily those of the institutions with which we are affiliated.

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1 Introduction

Investors worldwide have delegated the management of almost $100 trillion to specialized asset managers. One pervasive feature of the managers' compensation contracts is pay for performance relative to a benchmark.[1] While such benchmarking is very common in practice, it is not immediate why it is optimal to benchmark an asset manager. Surprisingly, benchmarking contracts in the asset management industry have attracted very little academic research and we intend to fill this gap. We ask the following questions: Why are benchmarks so pervasive? More importantly, what are the general-equilibrium effects of benchmarking?

To study these questions, we embed an optimal-contracting model into a general-equilibrium setting. We show that optimally-designed contracts for asset managers involve benchmarking. This is because conditioning asset managers' compensation on the performance of a benchmark portfolio partially protects them from risk, and thus gives them incentives to generate higher returns. In general equilibrium, the use of such incentive contracts creates a pecuniary externality through their effect on asset prices. Benchmarking inflates asset prices and reduces expected returns, thereby reducing the marginal benefit of using incentive contracts for others. We show that a social planner, who internalizes this externality, would opt for less incentive provision and less benchmarking.

Here is how our model works. Some agents in the economy, whom we call conventional investors, manage their own money and others, whom we call shareholders, delegate their investment choice to asset managers. All agents are risk averse. The asset managers' portfolios are unobservable to shareholders and (part of) the cost of managing a fund is private. The managers are paid based on incentive contracts designed by their shareholders. We focus on linear contracts which include a fixed salary, a fee for absolute performance, and potentially a fee for performance relative to a benchmark.

We assume that asset managers can generate superior returns relative to those of the conventional investors through various sophisticated strategies. These include lending securities, conserving on transactions costs (e.g., from crossing trades in-house or by obtaining favorable quotes from brokers) or providing liquidity (i.e., serving as a counterparty to liquidity demanders and earning a premium on such trades). While engaging in these activities augments returns, the asset manager has to incur a private cost in doing so. Shareholders

[1] For example, Ma, Tang, and Gómez (2019) report that around 80% of U.S. mutual funds explicitly base compensation on performance relative to a benchmark (usually a prospectus benchmark such as the S&P 500, Russell 2000, etc.).

need to incentivize the manager to take these actions but they cannot directly contract on them. A contract that pays the manager based on fund performance rewards the manager for taking these actions, but also exposes the manager to risk (because returns are stochastic). This risk, if unmitigated, means that the asset manager will under-invest in the return-augmenting activities. Adding a benchmark to the contract partially protects the manager from this risk and therefore will be used by shareholders to improve the incentives for the managers.

One consequence of the benchmarks is that they induce asset managers to engage in similar trades. Specifically, all managers invest more in stocks that are compatible with the sophisticated strategies and in stocks that are in their benchmarks. The managers' demand boosts prices of such stocks and lowers their expected returns. In other words, benchmarking contracts give rise to crowded trades.

Individual shareholders in our model take prices as given and they do not internalize the effects of contracts they design on equilibrium prices. Crowded trades resulting from the contract-induced incentives are in fact a pecuniary externality that reduces the effectiveness of contracts designed by other shareholders. Each manager still has to incur the full private cost of managing assets but the benefits of doing so are reduced because of the crowded trades. In such circumstances, a natural question to ask is how incentive contracts chosen by a social planner, who is subject to the same restrictions as individual shareholders but recognizes the effect of contracts on prices, differ from privately-optimal ones.

We show that individual shareholders underestimate the cost of incentive provision relative to the social planner, who internalizes the negative externality of incentive contracts. As a result, the planner opts for less incentive provision. Specifically, we show that both the absolute-performance sensitivity as well as the level of benchmarking are lower in the socially-optimal contract than in the privately-optimal one. This ameliorates the price pressure that asset managers exert and reduces the crowdedness of trades.

Our model can also contribute to the debate as to whether costs of asset management are excessive and whether returns delivered by the managers justify these costs. We do so by comparing the asset managers' costs and expected returns under privately- and socially-optimal contracts. We find that, from the socially-optimal point of view, individual shareholders excessively rely on contracts and make their asset managers invest too much at too high a cost.[2] In the equilibrium with privately-optimal contracts stock prices are too high and consequently expected per-share returns are lower that those under socially-optimal

[2]While the cost is borne by the asset managers, it ultimately gets passed on to the shareholders, who need to compensate the manager enough to ensure her participation.

contracts. Key to these implications is that, in contrast to individual shareholders, the social planner internalizes the pecuniary externality that gives rise to the crowded trades.

Finally, we investigate how benchmarks ought to be designed. We show that both privately- and socially-optimal benchmarks put more weight on stocks for which asset management adds more value as well as stocks for which incentive misalignment is most serious. The relative tilt in the weights, however, is different in the privately- and socially-optimal benchmarks. For example, the planner puts relatively less weight on stocks with large costs compared to individual shareholders. This is because the planner understands that the cost of incentive provision is effectively higher than individual shareholders perceive it to be, and is therefore less willing to use benchmark weights for incentive provision. We stress that our main results concern general-equilibrium consequences of contracts, which partial equilibrium analyses would necessarily miss.

The remainder of the paper is organized as follows. In the next section, we review the related literature. Section 3 presents our model, and Section 4 analyzes the model and derives our main results. Section 5 concludes and outlines suggestions for future research. Omitted proofs are in the Appendix.

2 Related Literature

Our work builds on the vast literature on optimal contracts under moral hazard, and in particular on seminal contributions of Holmstrom (1979) and Holmstrom and Milgrom (1987, 1991). Holmstrom argues that including in a contract a signal that is correlated with the output of the manager—in our case, such signal is the benchmark's performance—is beneficial to the principal. In our paper, the contract designer optimally chooses the signal to include in the contract. But more importantly, the benefit of including such signal is endogenous through the general-equilibrium effect on prices.

Holmstrom and Milgrom (1991) introduce a tractable contracting setting with moral hazard, with which our model shares many similarities. The standard implication in this literature is that increasing the agent's share in the output of a project (skin-in-the-game), increases the agent's exposure to the risk of the project, which in turn helps provide incentives to the agent. In the context of delegated asset management, however, giving the agent a larger share of portfolio return encourages her to scale down the risk of the (unobservable) portfolio by reducing risky asset holdings. Stoughton (1993) and Admati and Pfleiderer (1997) show that the manager is able to completely "undo" her steeper incentives by such scaling, and her incentives to collect information on asset payoffs remain unchanged. In our paper, we design a contract that provides desired incentives, despite the endogenous portfolio response of the manager, and show that it involves benchmarking. Another notable difference from the aforementioned literature is that we embed optimal contracts in a general-equilibrium setting and study interactions between contracts and equilibrium prices, and the implications of these interactions on welfare.

Our work is also related to the literature in asset pricing and corporate finance theory that explores the general-equilibrium implications of benchmarking. The pioneering work of Brennan (1993) shows that benchmarking leads to lower expected returns on stocks included in the benchmark. Cuoco and Kaniel (2011) and Basak and Pavlova (2013) study benchmarking in dynamic models, and show that the positive price pressure on benchmark stocks pushes up their prices and lowers their Sharpe ratios. Basak and Pavlova also show that benchmarking leads to excess volatility and excess co-movement of returns on stocks inside the benchmark. Kashyap, Kovrijnykh, Li, and Pavlova (2018) focus on implications of benchmarking asset managers on firm's corporate decisions and demonstrate that firms in the benchmark have a higher valuation for investment projects or merger targets than firms outside the benchmark. All this literature takes the benchmarking contract of asset managers to be exogenous. The only exceptions are Buffa, Vayanos, and Woolley (2014) and Cvitanic and Xing (2018), who study asset-pricing implications of benchmarking in an environment with endogenous contracts.[3] In both models, benchmarking helps reduce diversion of cash flows from the fund by asset managers. Our rationale for benchmarking is to reward activities that generate superior returns.

3 Model

We embed a linear optimal-contracting problem into a general-equilibrium asset-pricing framework. Each shareholder designs a compensation contract for his asset manager; in doing so, he takes equilibrium asset prices as given. In equilibrium, asset managers' demands, governed by their compensation contracts, end up affecting equilibrium prices. This, in turn, affects the effectiveness of individual contracts in the economy, as well as social

[3]See also Ozdenoren and Yuan (2017) who conduct a related analysis in the context of an industry equilibrium, in a classical moral-hazard setting with many principal-agent pairs. They show that benchmarking is privately-optimal but it creates overinvestment and excessive risk-taking at the industry level. Albuquerque, Cabral, and Guedes (forthcoming) present a related model of industry equilibrium, enriched further with strategic interactions among firms in the industry, and show that benchmarking against peer performance induces agents to take correlated actions.

welfare. The focus of our analysis is on the optimal compensation contracts and their equilibrium effects on prices and welfare.

3.1 Economy

Except for asset managers and their clients, our environment is standard. There are two periods, $t = 0, 1$. Investment opportunities are represented by N risky stocks and one risk-free bond. The risky stocks are claims to cash flows D, realized at $t = 1$, where $D \sim N(\mu, \Sigma)$. The variables D and μ are $N \times 1$ vectors and Σ is an invertible $N \times N$ matrix. The risk-free bond pays an interest rate that is normalized to zero. The risky assets are in a fixed supply of $\bar{x} > 0$ shares, where $\bar{x}$ is an $N \times 1$ vector. The bond is in infinite net supply. Let S, an $N \times 1$ vector, denote stock prices. Stock prices are determined endogenously in equilibrium.

There is a continuum of agents in the economy, of three types. First, there are "conventional" investors—constituting a fraction λ_C of the population—who manage their own portfolios. There are also asset managers—a fraction λ_{AM}—and shareholders who hire those asset managers—a fraction λ_S, with $\lambda_C + \lambda_{AM} + \lambda_S = 1$. We further assume for simplicity that each shareholder employs one asset manager, so that $\lambda_{AM} = \lambda_S$.[4] Shareholders can buy the bond directly, but cannot trade stocks, so they delegate the selection of their portfolios to asset managers. Each agent has a constant absolute risk aversion (CARA) utility function over final wealth (or compensation) W, $U(W) = -e^{-\gamma W}$, where $\gamma > 0$ is the coefficient of absolute risk aversion. Shareholders and conventional investors are endowed with x_{-1}^S and x_{-1}^C shares of stocks respectively, where $\lambda_S x_{-1}^S + \lambda_C x_{-1}^C = \bar{x}$.

For shareholders, delegating investment to an asset manager will have costs and benefits. On the one hand, as we will discuss in the next subsection, asset managers can potentially outperform conventional investors. On the other hand, the shareholder cannot dictate to the asset manager what portfolio she should choose. That is, the asset manager's portfolio choice is not contractible, or, equivalently, unobservable to the shareholder. It is true that some asset managers, e.g., mutual funds, are required to disclose their portfolios at a particular point in time. However, their actual portfolios between the disclosure dates differ significantly from their reported portfolios (Kacperczyk, Sialm, and Zheng, 2008), and a fund investor cannot obtain detailed information on the manager's trades. So the unobservability assumption is very realistic.

Furthermore, the asset managers incur a private cost of managing a fund. The combi-

[4] The extension where one asset manager is hired by multiple shareholders acting collectively is straightforward.

nation of the private cost and the portfolio choice being unobservable will be the central friction in our model.

We do not model an agent's choice to become a conventional investor or a shareholder—the fractions of different investors in the population are exogenous. One could endogenize this choice by assuming heterogeneous costs of participating in the stock market. We, however, abstract from this as it is not central to the main message of the paper.

3.2 Value-Added and Costs of Asset Management

We assume that asset managers can potentially outperform conventional investors. The (per-share) return for a conventional investor's portfolio x is given by $x^\top(D-S)$. The asset manager's returns are

$$r_x = x^\top(\Delta + D - S) + \varepsilon, \tag{1}$$

where $\Delta > 0$ is an (exogenous) vector and $\varepsilon \sim N(0, \sigma_\varepsilon)$ is a (scalar) noise term. The manager incurs a private portfolio-management cost $\psi^\top x$ where $\psi > 0$ is an exogenous vector.[5] So asset managers in our model incur costs to generate excess returns of $x^\top\Delta$ that we call α. The fact that this cost is private and the portfolio choice x is unobservable, will be the key driving force of our results.[6]

In this formulation, the α has nothing to do with the asset managers' stock-selection and market-timing abilities. If they did, then any conventional investors who happened to buy the same stocks or traded at the same time, without any knowledge of the Δ's, would earn the same returns. So this setup is consistent with the vast literature (e.g., Fama and

[5] Our assumption regarding managers' α-generating activities in (1) warrants further elaboration. Implicit in our expressions for the returns on the fund and the portfolio-management cost is that they scale linearly with the size of the portfolio. This seems to be inconsistent with Berk and Green (2004) who assume that there are decreasing returns to scale in asset management. There is no inconsistency. Berk and Green explicitly attribute decreasing returns to scale to the price impact of fund managers. The bigger the portfolio invested in an α-opportunity, the smaller the return on a marginal dollar invested. Berk and Green's model is in partial equilibrium and their price impact is simply an exogenous function of fund size. Ours is a general equilibrium model, in which asset managers collectively generate price impact, leading to lower expected returns of high-Δ stocks. The linearity of returns and costs in the size of the portfolio allows us to solve the model in closed form. We conjecture that our main results go through as long as the cost is strictly increasing in x. We are planning to incorporate this analysis in the next draft of the paper.

[6] One alternative formulation of the private cost that we have investigated assumes the asset manager needs to exert effort that is privately costly (and unobservable) to shareholder to generate the excess returns. That setup, which is often employed in the contracting literature (e.g. Holmstrom and Milgrom, 1987, 1991), yields a similar set of implications, though the algebra is more tedious. This equivalence gives us further confidence that the reduced-form choice that we have made is reasonable.

French, 2010) that casts doubt on the ability to generate abnormal returns by stock picking or market timing.

Instead, the asset managers' α comes from activities such as lending securities, delivering lower transactions costs (e.g., from crossing trades in-house or by obtaining favorable quotes from brokers) or providing liquidity (i.e., serving as a counterparty to liquidity demanders and earning a premium on such trades). We refer to these activities as "return-augmenting" strategies.

A critical assumption for our results is that the fund manager bears a private cost in delivering the abnormal returns. The existence of this cost is most clear-cut for the provision of liquidity. To successfully buy and sell at the appropriate times, the manager has to be actively monitoring market conditions while markets are open. For securities lending, the manager will also have to decide whether to accommodate requests to borrow shares. In some cases, these demands arise because the entity borrowing the shares wants to vote them and the manager must decide whether to pass up that choice. Adams, Mansi, and Nishikawa (2014) find that outsourcing the lending process to a third party yields lower returns than arranging this in-house. It seems reasonable that not all of these costs can be recouped. For the crossing of trades, we are less certain of how the internal decisions about this choice is made. Funds also report that client service requests tend to scale with the size of the portfolio and these costs are not fully recouped.[7]

The most powerful and convincing justification for the assumption that there are some private costs is that, if we accept the idea that some abnormal returns are obtainable, then it follows that you might naturally expect all asset managers to reap similar gains. Yet, the foregoing evidence suggests that there are major differences in the extent to which different managers pursue different strategies and thus earn different levels of revenue. If there were not some costs involved, then this heterogeneity would be very puzzling.

Turning to the revenues, there is a number of empirical papers that establish the profitability of securities lending, trading cost minimization and liquidity provision. We briefly review some of this evidence, explaining as we go why conventional investors would not be able to mimic asset managers to earn the same returns.

It is well-documented that securities lending is profitable. For example, it contributed 5% of total revenue of both BlackRock and State Street in 2017. Dimensional Fund Ad-

[7]Most managers also incur some costs that are observable and can be passed on directly to shareholders. Examples would include custody, audit, shareholder reports, proxies and some external legal fees. The analysis of the model where some costs are observable is available from the authors upon request and will be incorporated in the next draft of the paper.

visors' prospectuses report the net revenues from securities lending as a percentage of a portfolio's average daily net assets. There is tremendous variation across their different funds. For instance in 2018, the U.S. Large Company Portfolio earned 1 basis point, while the U.S. Small Cap Portfolio earned 11 basis points and the Emerging Markets Small Cap Portfolio earned 78 basis points. While it has recently become possible for some retail investors to participate in securities lending, they earn lower returns for this activity and do not have the same opportunities as a large asset manager.

It is also well-established that asset managers can profit from providing immediacy in trades, by either buying stocks which are out of favor or selling ones that are in high demand. In a classic paper, Keim (1999) estimates an annual α of 2.2% earned by liquidity provision activities of a fund. Rinne and Suominen (2016) document that mutual funds differ noticeably in the extent to which they either demand or supply liquidity. They estimate that the top decile of supplying funds outperform the bottom decile by about 60 basis points per year. Anand, Jotikasthira, and Venkataraman (2018) find similar estimates using a different sample of funds over a different time period. It would be prohibitively expensive for retail investors to try to do this.

While practitioners often mention the advantages that funds managers have in minimizing transactions costs, there is less empirical work quantifying these gains. One exception is Eisele, Nefedova, Parise, and Peijnenburg (forthcoming), which draws on transaction-level data that allows them to compare trades inside a fund complex with equivalent ones between outside parties. By crossing buy and sell orders, the affiliated transaction can save the bid-ask spread that would normally be paid. They find that indeed since the Securities and Exchange Commission began monitoring the within complex deals in 2004, the crossed trades are executed more cheaply than comparable external trades. Again conventional investors simply do not have the ability to follow this strategy.

Another modeling assumption is that the aforementioned benefits are proportional to the size of the holdings. For the liquidity provision and trade-crossing, this assumption strikes us as obvious. The wider the range of securities in the portfolio and/or the more a fund holds on any particular security, the easier it will be to provide liquidity or more likely it will be that a trade can be offset. For securities lending, by expanding the range of assets in the portfolio we would expect to open up additional lending opportunities. Whether holding more of any particular stock improves lending opportunities is less obvious. The demand for borrowing stocks or bonds varies greatly over time and across securities, so knowing whether having a large position will be valuable is uncertain.

Finally, the noise term ε in (1) captures the fact that aforementioned return-augmenting activities do not produce a certain return each period. For example, the demand for liquidity, the opportunities to lend share and the possibility of crossing-trades all fluctuate so even a very alert and skilled manager will have some randomness in their returns. Also for securities that are lent, there is a risk that they will not be returned in a timely manner or potentially at all.[8]

3.3 Asset Managers' Compensation Contracts

To provide incentives for the asset managers to invest into the risky stocks and to generate α, the shareholders design compensation contracts. The asset managers receive compensation w from shareholders. This compensation has three parts: one is a linear payout based on absolute performance of the manager's portfolio x, the second part depends on the performance relative to the benchmark portfolio, and the third is independent of performance.[9] Then

$$w = \tilde{a} r_x + b(r_x - r_{\mathbf{b}}) + c = a r_x - b r_{\mathbf{b}} + c, \tag{2}$$

where r_x is the performance of the manager's portfolio defined in (1), $r_{\mathbf{b}} = \theta^\top (D - S)$ is the performance of the benchmark portfolio θ, and $a = \tilde{a} + b$ is a redefined coefficient, which we will explain below. The contract is $(\tilde{a}, b, c, \theta)$—or, equivalently, (a, b, c, θ)—where $\tilde{a}$ is the fee for absolute performance, b is the fee linked to relative performance, c is the fixed component, and θ is the $N \times 1$ vector of benchmark weights. All of our analysis and the intuitions that follow will be in terms of a, which we will refer to as "skin-in-the-game" or absolute-performance sensitivity. The contract for a particular asset manager is optimally chosen by the shareholder who employs her.

The restriction of linearity of optimal contracts warrants a discussion. We assume linearity for the purpose of tractability. Characterizing fully-optimal contracts is hard in general.[10] It is even harder in our case since we solve for them in a general-equilibrium model where contracts affect equilibrium prices and thus in turn affect the contract chosen by each shareholder. The restriction to linearity allows us to find optimal contracts in

[8] One might wonder what happens of the noise is proportional to x (that is, the noise term is εx instead of ε). The algebra is messier in this case, but the main results go through. The analysis of this case is available from the authors upon request.

[9] This part captures features such as a fee linked to initial assets under management or a fixed salary or any other fixed costs

[10] In the famous paper, Holmstrom and Milgrom (1987) show that, under specific conditions, the solution for the optimal contract in a dynamic environment is as if the problem were the static one and the principal is constrained to use a linear compensation rule. The assumptions of this paper are not satisfied in our setting, though.

closed form.

We think of an asset manager's contract as a compensation contract between a portfolio manager and her investment-advisor firm (e.g., BlackRock.) For U.S. mutual funds, we have a great deal of information about such contracts. Since 2005 mutual funds in the U.S. have been required to include a "Statement of Additional Information" in the prospectus that describes how portfolio managers are compensated. Ma, Tang, and Gómez (2019) analyze this information and find that around 80% of the funds explicitly base compensation on performance relative to a benchmark (usually the prospectus benchmark, e.g., S&P 500, Russell 2000, etc.). Some funds state in their Statements of Additional Information that managers are encouraged to invest in their own funds, i.e., have skin-in-the-game. Khorana, Servaes, and Wedge (2007) document mutual fund managers' holdings of their own funds, which provides evidence for an absolute performance component in their compensation. Ma, Tang, and Gómez report that most managers also have a fixed salary component, but the fraction of fund managers whose entire compensation consists of only fixed salary is very small. The performance-based bonus exceeds the fixed salary for 68% of the funds in the sample, constituting more than 200% of fixed salary for 35% of funds.[11] Evidence on compensation contracts of asset managers other than mutual funds is sparse because these managers are not required to report their compensation structure and they rarely do so voluntarily. Bank for International Settlements (2003) presents survey-based evidence for a sample of other asset managers including sovereign wealth funds and pension funds, and also finds evidence supporting our compensation structure, and in particular finds that performance evaluation relative to benchmarks is pervasive.

4 Analysis

4.1 Conventional Investors' and Asset Managers' Problems

At $t = 0$, conventional investors choose a portfolio of stocks x and the bond holdings to maximize their expected utility $-Ee^{-\gamma W}$. Since their return on the portfolio is $x^\top(D - S)$, the resulting time-1 wealth is $W = \left(x^C_{-1}\right)^\top S + x^\top(D - S)$. It is well-known that a conventional investor's maximization problem is equivalent to the following mean-variance

[11]In contrast, Ibert, Kaniel, Van Nieuwerburgh, and Vestman (2017) find surprisingly weak sensitivity of manager pay to performance for Swedish mutual funds.

optimization:

$$\max_{x} x^{\top}(\mu - S) - \frac{\gamma}{2} x^{\top} \Sigma x. \tag{3}$$

Asset managers choose a portfolio of stocks x and the bond holdings to maximize $-E\exp\{-\gamma[ar_x - br_{\mathbf{b}} + c - x^{\top}\psi]\}$, where the quantity inside the square brackets is their compensation net of private cost. This maximization problem is equivalent to the following optimization:

$$\max_{x} \quad x^{\top}(a\Delta - \psi) + (ax - b\theta)^{\top}(\mu - S) + c - \frac{\gamma}{2}\left[(ax - b\theta)^{\top}\Sigma(ax - b\theta) + a^2\sigma_{\varepsilon}^2\right], \tag{4}$$

where we have substituted r_x and $r_{\mathbf{b}}$ defined above. The first three terms are the manager's expected pay, net of her private cost, and the last term is the variance of her compensation, scaled by a half of her absolute risk aversion γ. One important observation we make at this stage is related to the first term in (4): the manager receives a fraction a of the per-share abnormal return on the stocks, Δ, but pays the entire cost ψ per share. (We later show that $a < 1$.)

Both the conventional investors and asset managers take asset prices as given. Lemma 1 reports the optimal portfolios of the conventional investors and asset managers arising from their optimizations.

Lemma 1 (**Portfolio Choice**). *The conventional investors' and asset managers' portfolio demands are as follows:*

$$x^{C} = \Sigma^{-1}\,\frac{\mu - S}{\gamma}, \tag{5}$$

$$x^{AM} = \Sigma^{-1}\,\frac{\Delta - \psi/a + \mu - S}{a\gamma} + \frac{b\theta}{a}. \tag{6}$$

A conventional investor's portfolio is the standard mean-variance portfolio, scaled by his risk aversion γ. An asset manager's portfolio differs from that of a conventional investor in three dimensions. First, asset managers split their risky assets investments between two portfolios: the (modified) mean-variance portfolio and the benchmark portfolio. The latter arises because the manager's compensation is exposed to fluctuations in the benchmark. She desires to hedge this exposure and holds a hedging portfolio that is (perfectly) correlated with the benchmark, i.e., the benchmark itself. This implication is very general, and we share it with other models that analyzed benchmarking, both in two-period and multiperiod economies and for other investor preferences specifications.[12] The split between the two portfolios is governed by the strength of the relative-performance incentives, captured by b. The higher b is, the closer the manager's portfolio is to the benchmark.

Second, because our managers have access to return-augmenting strategies, they perceive the mean-variance tradeoff differently from the conventional investors and tilt their mean-variance portfolios towards high-Δ stocks. Consistent with this result, Johnson and Weitzner (2019) report that asset managers' portfolios in their sample overweight stocks with high securities-lending fees.

Finally, the asset manager scales her (modified) mean-variance portfolio by $1/a$ relative to that of a conventional investor. The reason for the scaling is that, as we can see from the first term in (2), for each share that an asset manager holds, she gets a fraction a of the total return.

Let us now focus on the variance of the manager's compensation, $(ax^{AM} - b\theta)^\top \Sigma(ax^{AM} - b\theta) + a^2\sigma_\varepsilon^2$, which we have first encountered in (4), and compare two managers, one of whom does not have a benchmark ($b = 0$) and one does ($b > 0$). Absent benchmarking, the safest investment choice for the manager is to invest everything in the riskless asset ($x^{AM} = 0$). With benchmarking, the safest investment choice is to hold the benchmark portfolio ($x^{AM} = b\theta/a$), or to be a "closet indexer." Both managers would like to take some risk (and earn the corresponding return), and they do so by investing in the mean-variance portfolio. For the same skin-in-the-game a, the benchmarked manager is willing to hold $b\theta/a$ *more shares* in the risky assets. The variances of the compensations of the two managers are exactly the same.[13] This is the sense in which benchmarking partially protects the manager from risk.

We are now ready to solve for equilibrium prices. Substituting portfolio demands from Lemma 1 into the market-clearing condition for stocks, $\lambda_{AM}x^{AM} + \lambda_C x^C = \bar{x}$, we find

$$S = \mu - \gamma\Sigma\Lambda\left(\bar{x} - \lambda_{AM}\frac{b\theta}{a}\right) + \Lambda\frac{\lambda_{AM}}{a}\left(\Delta - \frac{\psi}{a}\right), \tag{7}$$

where

$$\Lambda \equiv \left[\frac{\lambda_{AM}}{a} + \lambda_C\right]^{-1} \tag{8}$$

modifies the market's effective risk aversion.

[12] This result first appears in Brennan (1993) in a two-period model. Cuoco and Kaniel (2011) and Basak and Pavlova (2013), among others, obtain it in dynamic models with different preferences.

[13] To see this formally, note that $ax^{AM} - b\theta = \Sigma^{-1}(\Delta - \psi/a + \mu - S)/\gamma$ does not depend on b.

To develop intuition, it is useful to compare the stock prices in our economy (7) with their counterparts in the economy with conventional investors only ($\lambda_{AM} = 0,\ \lambda_C = 1$). In that economy $S = \mu - \gamma \Sigma \bar{x}$, a familiar expression. We can see that the asset managers, who have an additional demand for benchmark stocks and for high-Δ stocks, push prices of such stocks up (relative to otherwise identical low-benchmark-weight and low-Δ stocks). Specifically, the term $\gamma \Sigma \Lambda \lambda_{AM} b\theta / a$ in (7) accounts for the price pressure due to benchmarking and the term $\Lambda(\lambda_{AM}/a)(\Delta - \psi/a)$ for α-chasing. In our economy, higher prices imply lower expected returns. Therefore, the asset managers collectively exert price impact and lower expected returns on the stocks that they particularly desire to hold in their portfolios.[14]

4.2 Shareholders' Problem

Each shareholder chooses the contract (a, b, c, θ) and portfolio $x = x^{AM}$ to maximize his expected utility

$$-E \exp\left\{-\gamma\left[\left(x_{-1}^S\right)^\top S + r_x - (ar_x - br_{\mathbf{b}}) - c\right]\right\}$$

subject to the asset manager's participation constraint

$$-E \exp\left\{-\gamma\left[(ar_x - br_{\mathbf{b}}) + c - x^\top \psi\right]\right\} \geq u_0, \tag{9}$$

and her incentive constraint (6). The latter is the first-order condition of the asset manager's optimization problem, capturing the fact that the portfolio x is the asset manager's private choice. The right-hand side of the participation constraint, u_0, is the value of asset manager's outside option.[15]

Equivalently, we can rewrite the shareholder's problem in terms of mean-variance utilities. It will be convenient to express payoffs in terms of the following variable. Denote $y = ax^{AM} - b\theta$ and $z = x^{AM} - y$, which are effective allocations of stock holdings to the asset manager and shareholder, respectively. Furthermore, define the shareholder's and

[14]Note that this discrepancy cannot be arbitraged away in our model. Conventional investors are free to engage in any arbitrage activity investors because the are unrestricted in their portfolio choice. As long as asset managers represent a meaningful fraction of the market (i.e., λ_{AM} is non-negligible), however, there are always differences in prices of stocks that depend on their benchmark weights and asset managers' Δ.

[15]We do not model explicitly what this outside option is, as it does not matter for our main results. It can be exogenous, or it can be endogenized.

asset manager's mean-variance utilities as

$$U^S\left(a, \frac{b\theta}{a}, y, S\right) = \left(x^{AM}\right)^\top (1-a)\Delta + z^\top(\mu - S) - \frac{\gamma}{2}\left[z^\top \Sigma z + (1-a)^2\sigma_\varepsilon^2\right] + \left(x_{-1}^S\right)^\top S - c,$$

$$U^{AM}(a, y, S) = \left(x^{AM}\right)^\top (a\,\Delta - \psi) + y^\top(\mu - S) - \frac{\gamma}{2}\left[y^\top \Sigma y + a^2\sigma_\epsilon^2\right] + c,$$

where

$$x^{AM} = \frac{y}{a} + \frac{b\theta}{a}, \tag{10}$$

$$z = \frac{1-a}{a}y + \frac{b\theta}{a}. \tag{11}$$

Then the shareholder's problem is to maximize his utility subject to the asset manager's participation constraint, and her (modified) incentive constraint,

$$\max_{a,b,c,\theta,y} \; U^S \tag{12}$$

$$\text{s.t. } U^{AM} \geq \tilde{u}_0, \tag{13}$$

$$y = \Sigma^{-1}\frac{\Delta - \psi/a + \mu - S}{\gamma}, \tag{14}$$

where the value $\tilde{u}_0$ is the mean-variance version of u_0.[16]

Notice that because of the contract's constant component c, in the mean-variance formulation utility becomes transferable, and the shareholder effectively maximizes the total utility of the shareholder and asset manager subject to the asset manager's incentive constraint. The asset manager's participation constraint is then trivially satisfied by adjusting the constant c.

We next discuss the roles that the contract parameters a, b, and θ play in the shareholder's maximization problem.

4.3 Contracts and Incentives

As a point of reference, consider the first best where the asset manager's portfolio choice is observable and contractible. The first-best contract involves perfect risk sharing between the (equally risk-averse) shareholder and asset manager and no benchmarking, $a = 1/2$ and $b = 0$.[17] It is easy to show that if the asset manager were facing the first-best contract

[16]In particular, if the asset manager's outside option is risk-free, then $u_0 = -\exp(-\gamma\tilde{u}_0)$.

[17]See Lemma 5 in the Appendix for the formal analysis.

but chose the portfolio privately, she would overestimate the cost of α-production, thus investing less in all stocks, especially stocks with high ψ. A higher a reduces the asset manager's effective cost ψ/a, which increases her demand for risky assets, especially those with a high cost of α-production. However, a higher a also exposes the asset manager to more risk, which makes her scale down x^{AM}, as reflected in the denominator(s) of (10). Thus the use of absolute performance creates a tension between incentive provision and risk sharing.

The use of benchmarking, together with an appropriate benchmark selection, alleviates this tension by mitigating the second, adverse effect of a. As we have discussed in Section 4.1, benchmarking shields the asset manager from risk by reducing variance in her compensation *for a given portfolio choice*.[18] As a result, (for the same a) the asset manager invest more. In addition, if the benchmark portfolio puts a relatively higher weight on certain stocks, the manager's exposure to risk is reduced more for those stocks, and she will invest proportionally more in them. That is, benchmarking protects the asset manager from risk, and an appropriate choice of the benchmark portfolio can help to improve incentives for α-production.

4.4 Privately-Optimal Contracts

Notice that the shareholder fully internalizes the asset manager's cost of managing the fund. Formally, this can be seen by taking the first-order condition with respect to c, which immediately implies that the Lagrange multiplier on the participation constraint equals one. But since the asset manager bears the cost privately and only receives fraction a of the return, for her the effective cost is higher, which is why ψ/a appears in (6). These two costs being different plays an important role in our analysis.

All the main tradeoffs will be apparent from the first-order conditions. It will also be useful to compare the first-order conditions of an individual shareholder to those of a social planner. Therefore in what follows, we present and carefully discuss those conditions.

Notice that b enters into the shareholder's and asset manager's problems only though $b\theta/a$. Therefore we take the first-order condition with respect to $b\theta/a$, and later derive the expression for b separately. The first-order condition with respect to $b\theta/a$ is given by

$$\frac{\partial(U^S + U^{AM})}{\partial(b\theta/a)} = \Delta - \psi + \mu - S - \gamma\Sigma z = 0. \tag{15}$$

[18] By reducing in the asset manager's risk exposure, benchmarking makes it cheaper for the shareholder to implement any particular portfolio choice.

It captures the marginal effect on the total utility of the shareholder and asset manager due to a higher demand by the asset managers in response to more benchmarking. Substituting (11) and (14) into the above equation and rearranging terms (see the Appendix), we have

$$\gamma\Sigma b\theta = (2a-1)\left(\Delta - \psi + \mu - S\right) + (1-a)\left(\frac{1}{a} - 1\right)\psi. \tag{16}$$

The two terms in equation (16) capture two concerns that shareholders have in mind when designing the benchmark. Consider two special cases, the first one is $a = 1/2$ when perfect risk-sharing is achieved, and the second one is $a = 1$ when the private and social costs are aligned. As we will show later, in the optimal contract $a \in (1/2, 1)$, so both terms on the right-hand side of (16) are positive. The first term, $(2a-1)\left(\Delta - \psi + \mu - S\right)$, is there because the shareholder recognizes that benchmarking increases the total expect surplus net of cost. Since $a > 1/2$, the asset manager is exposed to more risk than unconstrained optimal, so the shareholder uses benchmarking to make her invest more, in particular in stocks with a higher value added $\Delta - \psi$. The second term, $(1-a)(1/a - 1)\psi$, reflects the incentive-provision role of $b\theta$. By protecting the asset manager from risk, benchmarking provides her with incentives to invest more. Such incentive provision is especially important for stocks with high cost ψ, as the asset manager is the most reluctant to invest in them.

Let us now examine the first-order condition with respect to a, which is given by[19]

$$\begin{aligned} 0 &= \frac{\partial(U^S + U^{AM})}{\partial a} + \frac{\partial U^S}{\partial y}\frac{\partial y}{\partial a} \\ &= -(2a-1)\gamma\sigma_\epsilon^2 - \left(\Delta - \psi + \mu - S - \gamma\Sigma z\right)^\top \frac{y}{a^2} + \frac{1-a}{a}\left(\Delta + \mu - S - \gamma\Sigma z\right)^\top \frac{\partial y}{\partial a} \\ &= -(2a-1)\gamma\sigma_\epsilon^2 + \frac{1-a}{a}\psi^\top \frac{\partial y}{\partial a}, \end{aligned} \tag{17}$$

where the last equality follows from (15). Notice the appearance of $\partial y/\partial a$ in the above equation. It captures how the surplus is affected through a direct response of the asset manager's (modified) demand for the risky assets, y, to a marginal increase in a. This is the incentive-provision channel. The other terms reflect that the design of a also affects how risk is split between the shareholder and the asset manager (and thus also how much of the risky asset the asset manager buys).

Using (15) and $\partial y/\partial a = \Sigma^{-1}\psi/(\gamma a^2)$ (obtained by differentiating (14) with respect to

[19]Recall that the asset manager's utility is maximized with respect to y, so $(\partial U^{AM}/\partial y)(\partial y/\partial a)$ does not appear in (17).

a), the above equation becomes

$$(1-a)\frac{\psi^\top \Sigma^{-1}\psi}{\gamma a^3} - (2a-1)\gamma\sigma_\varepsilon^2 = 0. \tag{18}$$

Notice again the terms with $(1-a)$ and $(2a-1)$, except unlike in (16), the terms now have different signs. This means that there is a tradeoff that these two terms represent. A higher a is beneficial as it provides incentives for α-production, which is captured by the first term, but is also costly as it exposes the asset manager to too much risk, as captured by the second term.

Substituting the expression for S, we obtain closed-form expressions for equilibrium contracts given in part (a) of the next lemma.

Lemma 2. *In the equilibrium with the privately-optimal contract,*

(a) $a = a^{private}$, $b = b^{private}$, *and* $\theta = \theta^{private}$ *are given by*

$$0 = (1-a)\frac{\psi^\top \Sigma^{-1}\psi}{\gamma a^3} - (2a-1)\gamma\sigma_\varepsilon^2, \tag{19}$$

$$b = (2a-1)\,\mathbf{1}^\top\left[\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_C(\Delta-\psi)\right] + (1-a)\left[\frac{1}{a} - \left(\frac{\lambda_{AM}}{a} + \lambda_C\right)\right]\mathbf{1}^\top\frac{\Sigma^{-1}}{\gamma}\psi, \tag{20}$$

$$\theta = \frac{2a-1}{b}\left[\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_C(\Delta-\psi)\right] + \frac{1-a}{b}\left[\frac{1}{a} - \left(\frac{\lambda_{AM}}{a} + \lambda_C\right)\right]\frac{\Sigma^{-1}}{\gamma}\psi; \tag{21}$$

(b) asset prices are given by

$$S^{private} = \mu - \gamma\Sigma\bar{x} + \lambda_{AM}\left(\Delta - \psi + \Delta - \frac{\psi}{a}\right) \tag{22}$$

and the asset managers' asset holdings are

$$x_{AM}^{private} = 2\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_C\left(\Delta - \psi + \Delta - \frac{\psi}{a}\right). \tag{23}$$

Notice that (19) is the expression in a only. Then given a, (20) delivers the expression for b, and finally, given a and b, (21) gives us expression for the benchmark weights θ. We will use (20) to provide sufficient conditions for the shareholder to use benchmarking in what follows. Before we proceed with that, let us briefly comment on the expression for the equilibrium prices given by (22). Recall that in the economy without asset managers the equilibrium prices would be $S = \mu - \gamma\Sigma\bar{x}$. Prices are higher in the presence of asset managers due to their higher demands as engage in return-augmenting activities, as captured by the last term in (22). Notice that it contains $\Delta - \psi$ and $\Delta - \psi/a$, which are the extra expected returns net of costs as perceived by the shareholders and by the asset managers, respectively. Similarly, the equilibrium asset holdings of asset managers in (23) are higher when the opportunities for α-production are better. Notice that asset managers hold exactly $2\bar{x}$ when $\lambda_C = 0$. We will discuss this special case further in subsection 4.5.

For some of our results on benchmarking, we will need to impose the following assumption.

Assumption 1. *Suppose that*

(a) $\mathbf{1}^\top \left[\bar{x} + \lambda_C \Sigma^{-1}(\Delta - \psi)/\gamma\right] > 0$,

(b) $\mathbf{1}^\top \Sigma^{-1}\psi > 0$.

Assumption 1 is a mild technical restriction. It is trivially satisfied when the variance-covariance matrix Σ is diagonal or when Δ's and ψ's are the same for all stocks (and given that $\Delta - \psi \geq 0$). When Σ is not diagonal (which implies that cross-price elasticities of the asset manager's demand function are not zero), it is useful to interpret the assumption as follows. Part (a) is a necessary and sufficient condition for the sum of shares (over all assets) that the asset manager holds in the first best to be positive (that is, the asset manager does not borrow).[20] Part (b) means that if the private cost ψ increases by the same percentage for all assets, the sum of shares (over all assets) that the asset manager holds in equilibrium goes down. In other words, the asset manager reduces total holdings when the cost is higher.

Using the above assumption and the equilibrium expression for b presented in Lemma 2, we have the following result:

Proposition 1 (Benchmarking is Optimal). *Suppose that Assumption 1 holds. Then the privately-optimal contract involves benchmarking, that is,* $b^{private} > 0$.

Proposition 1 helps us understand why benchmarking in the asset management industry is so pervasive. The key theoretical insight that it provides is that shareholders are better off and could incentivize the manager to engage more in risky α-generating activities if they partially protect her from risk by benchmarking. In the language of the asset management

[20]See the proof of Lemma 5 in the Appendix.

industry, benchmarked managers are being protected from "β" (i.e., the fluctuations in the return of the benchmark portfolio) while being rewarded for α.

Next, we discuss the properties of the privately-optimal benchmark weights. Using equation (21), the lemma below shows how these weights differ across stocks with different value added or cost of α-production, which are $\Delta - \psi$ and ψ respectively.

Lemma 3. *Consider two stocks, i and j, that have the exact same characteristics except $\Delta_i - \psi_i \geq \Delta_j - \psi_j$ and $\psi_i \geq \psi_j$, with at least one inequality being strict. Then in the privately-optimal contract, stock i has a larger weight in the benchmark than stock j: $\theta_i^{private} > \theta_j^{private}$.*

The reason for this result is intuitive—shareholders recognize that manipulating benchmark weights allows them to provide more incentives for investment in stocks where α-production is the most valuable. The effect of a larger ψ on the benchmark weight is ambiguous, as can be seen from (21). On the one hand, the incentive problem is the most severe for stocks with a larger ψ, and thus setting higher weight is most valuable for those stocks. On the other hand, a larger ψ reduces the total expected return, which reduces the marginal benefit of using $b\theta$ for protecting the asset manager from extra risk. However, for the same (or a larger) value added, higher-cost stocks would have a higher weight in the privately-optimal benchmark.

Shareholders design contracts to influence the asset manager's demand for risky assets. Through the aggregate demand of the asset managers, contracts influence equilibrium asset prices, as given by (7). Prices then affect the marginal cost/marginal benefit tradeoff of contracts for individual shareholders. Since shareholders take prices as given, they do not internalize how their choices of contracts (once aggregated) affect effectiveness of other shareholders' contracts. In other words, shareholders impose an externality on each other through their use of contracts. It is natural to wonder then what the contract would be if chosen by a social planner, who is subject to the same restrictions as shareholders, but internalizes the effect of contracts on prices. We explore this issue in the next subsection.

4.5 Socially-Optimal Contracts

We define the problem of such a constrained social planner as follows. The planner maximizes the weighted average of shareholders' and conventional investors' utilities subject to the participation and incentive constraints of the asset managers, as well as the constraint that conventional investors choose their portfolios themselves.[21] As before this problem can be equivalently rewritten in terms of mean-variance preferences.[22] Specifically, define

$$U^C = \left(x_{-1}^C\right)^\top S + \left(x^C\right)^\top (\mu - S) - \frac{\gamma}{2}\left(x^C\right)^\top \Sigma x^C.$$

Then the social planner's problem is

$$\max_{a,b,c,\theta,y} \ \omega_S U^S + \omega_C U^C$$

subject to (13), (14), and (5).

The social planner's first-order condition with respect to $b\theta/a$ is

$$\begin{aligned} 0 = &\left[\omega_S\left(x_{-1}^S - x^{AM}\right)^\top + \omega_C\left(x_{-1}^C - x^C\right)^\top\right]\frac{\partial S}{\partial(b\theta/a)} \\ &+ \omega_S\left[\frac{\partial(U^S + U^{AM})}{\partial(b\theta/a)} + \frac{\partial U^S}{\partial y}\frac{\partial y}{\partial S}\frac{\partial S}{\partial(b\theta/a)}\right]. \end{aligned} \tag{24}$$

The terms in the first line of the above equation capture what we call the redistribution effect. Depending on the initial endowments and the Pareto weights, the social planner has incentives to use benchmarking to move prices so as to benefit one or the other party based on this redistribution motive. We discuss the redistribution effects in Remark 1 at the end of this subsection. To isolate the effects of benchmarking not coming from this redistribution motive, which we find more interesting as they capture the actual externality, we set the Pareto weights $\omega_S = \omega_C$. Then by market clearing, $\omega_S\left(x_{-1}^S - x^{AM}\right) + \omega_C\left(x_{-1}^C - x^C\right) = 0$, the term in the first line of (24) is zero.[23] Rewriting the term in the second line, (24) becomes

$$\begin{aligned} 0 &= (\Delta - \psi + \mu - S - \gamma\Sigma z)^\top + \frac{1-a}{a}(\Delta + \mu - S - \gamma\Sigma z)^\top \frac{\partial y}{\partial S}\frac{\partial S}{\partial(b\theta/a)} \\ &= (\Delta - \psi + \mu - S - \gamma\Sigma z)^\top - \frac{1-a}{a}(\Delta + \mu - S - \gamma\Sigma z)^\top \Lambda\lambda_{AM}, \end{aligned} \tag{25}$$

[21]Equivalently, instead of imposing the asset manager's participation constraint, her utility can be included into the planner's objective function with a Pareto weight ω_{AM}. For the transfer c to be finite, we must have $\omega_{AM} = \omega_S$. This is analogous to noticing that the Lagrange multiplier on the participation constraint, which effectively acts as the Pareto weight on the asset manager, equals ω_S.

[22]We provide the original formulation in terms of exponential utilities in the Appendix.

[23]Choosing Pareto weights to cancel out the redistribution effects is equivalent to allowing the social planner to use transfers. The planner will then use transfers to equate the marginal utilities (weighted by Pareto weights) of different agents.

or, equivalently,

$$\Delta - \psi \frac{\lambda_{AM}/a + \lambda_C}{\lambda_{AM} + \lambda_C} + \mu - S - \gamma \Sigma z = 0. \tag{26}$$

Compare (25) or (26) with (15). The second term in (either line of) (25) captures the general-equilibrium externality that the planner is trying to correct, and it is negative. The planner realizes that benchmarking inflates prices and thus reduces returns. Hence for the social planner the benefit of α-production is smaller due to this crowded-trades effect, or, equivalently, the cost is higher for the same unit of benefit: $\psi(\lambda_{AM}/a+\lambda_C)/(\lambda_{AM}+\lambda_C) > \psi$ in (26) instead of ψ in (15). So from the social planner's point of view, α-chasing is less beneficial/more expensive, which, as we will see, will make her do less of it in equilibrium.

Substituting the expression for z,

$$\gamma \Sigma b\theta = (2a-1)\left(\Delta - \frac{\lambda_{AM}/a + \lambda_C}{\lambda_{AM} + \lambda_C}\psi + \mu - S\right) + (1-a)\left(\frac{1}{a} - \frac{\lambda_{AM}/a + \lambda_C}{\lambda_{AM} + \lambda_C}\right)\psi. \tag{27}$$

Again, compared to (16), the social cost is inflated.

The planner's first-order condition with respect to a (after again canceling out the redistribution effects) is

$$\begin{aligned} 0 &= \frac{\partial(U^S + U^{AM})}{\partial a} + \frac{\partial U^S}{\partial y}\left[\frac{\partial y}{\partial a} + \frac{\partial y}{\partial S}\frac{\partial S}{\partial a}\right] \\ &= (1-2a)\gamma\sigma_\epsilon^2 - (\Delta - \psi + \mu - S - \gamma\Sigma z)^\top \frac{y}{a^2} + \frac{1-a}{a}(\Delta + \mu - S - \gamma\Sigma z)^\top \left[\frac{\partial y}{\partial a} + \frac{\partial y}{\partial S}\frac{\partial S}{\partial a}\right]. \end{aligned}$$

Comparing this to (17), there is an additional term containing $(\partial y/\partial S)(\partial S/\partial a)$. It reflects that the planner understands that contracts affect prices, which in turn affect the asset managers' demands and thus the marginal benefit of α-production. However, unlike in the first-order condition with respect to $b\theta/a$, we cannot sign this extra term—recall that the effect of a on the asset manager's incentives is ambiguous. That is, *for a given $b\theta/a$*, the social planner's benefit of using a can be higher or lower than that of an individual shareholder. What is interesting is that once the planner takes into account the adjustment in the social optimal $b\theta$, the effect of a that reduces x^{AM} and thus lowers prices is exactly offset by this adjustment. Hence the additional term that remains in the first-order condition with respect to a is only the part that takes into account how a higher a increases incentives for investment, which in turn increases prices and reduces returns. As a result, the marginal benefit of a for the social planner is lower than that for individual shareholders, and the possibility of benchmarking is a crucial for this result.

To show this formally, use (25) to rewrite the above equation as follows:[24]

$$
\begin{aligned}
0 &= -(2a-1)\gamma\sigma_\epsilon^2 + \frac{1-a}{a}\frac{\lambda_{AM}/a+\lambda_C}{\lambda_{AM}+\lambda_C}\psi^\top\left[\frac{\partial y}{\partial a} + \frac{\partial y}{\partial S}\frac{\partial S}{\partial a} + \frac{y}{a^2}\frac{\partial y}{\partial S}\frac{\partial S}{\partial(b\theta/a)}\right] \\
&= -(2a-1)\gamma\sigma_\epsilon^2 + \frac{1-a}{a}\frac{\lambda_{AM}/a+\lambda_C}{\lambda_{AM}+\lambda_C}\psi^\top\left[\frac{\partial y}{\partial a} - \frac{\partial y}{\partial a}\frac{\lambda_{AM}/a}{\lambda_{AM}/a+\lambda_C}\right].
\end{aligned}
$$

We can see that the effectiveness of incentive provision for the planner, captured by the term proportional to $\partial y/\partial a$, is smaller than for private shareholders in equation (18). Finally, using $\partial y/\partial a = \Sigma^{-1}\psi/(\gamma a^2)$, we obtain the analog of (18),

$$
(1-a)\frac{\psi^\top\Sigma^{-1}\psi}{\gamma a^3}\frac{\lambda_C}{\lambda_{AM}+\lambda_C} - (2a-1)\gamma\sigma_\varepsilon^2 = 0. \tag{28}
$$

The benefit of incentive provision captured by the first term is smaller than the corresponding term in (18). Comparing (28) with (18), it is easy to see that the social planner will use a lower a than individual shareholders. We will formalize this result later in Proposition 2.

Substituting for the equilibrium prices, the following Lemma described the equilibrium contract and prices in closed form.

Lemma 4. *In the equilibrium with the socially-optimal contract,*

(a) $a = a^{social}$, $b = b^{social}$ *and* $\theta = \theta^{social}$ *are given by*

$$
0 = (1-a)\frac{\psi^\top\Sigma^{-1}\psi}{\gamma a^3}\frac{\lambda_C}{\lambda_{AM}+\lambda_C} - (2a-1)\gamma\sigma_\varepsilon^2, \tag{29}
$$

$$
b = (2a-1)\mathbf{1}^\top\left[\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_C(\Delta-\psi)\right] + (1-a)\left[\frac{1}{a} - \frac{\lambda_{AM}/a+\lambda_C}{\lambda_{AM}+\lambda_C}\right]\mathbf{1}^\top\frac{\Sigma^{-1}}{\gamma}\psi, \tag{30}
$$

$$
\theta = \frac{2a-1}{b}\left[\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_C(\Delta-\psi)\right] + \frac{1-a}{b}\left[\frac{1}{a} - \frac{\lambda_{AM}/a+\lambda_C}{\lambda_{AM}+\lambda_C}\right]\frac{\Sigma^{-1}}{\gamma}\psi; \tag{31}
$$

(b) asset prices are given by

$$
S^{social} = \mu - \gamma\Sigma\bar{x} + \lambda_{AM}\left(\Delta - \frac{\lambda_{AM}/a+\lambda_C}{\lambda_{AM}+\lambda_C}\psi + \Delta - \frac{\psi}{a}\right) \tag{32}
$$

[24] To get the second line, differentiate the market-clearing condition $\lambda_{AM}(y/a + b\theta/a) + \lambda_C x^C = 0$ with respect to $b\theta/a$ and a and use $\partial x^C/\partial S = \partial y/\partial S$ to get $\left(\frac{\lambda_{AM}}{a}+\lambda_C\right)\frac{\partial y}{\partial S}\frac{\partial S}{\partial(b\theta/a)} + \lambda_{AM} = 0$ and $\left(\frac{\lambda_{AM}}{a}+\lambda_C\right)\frac{\partial y}{\partial S}\frac{\partial S}{\partial a} - \lambda_{AM}\frac{y}{a^2} + \frac{\lambda_{AM}}{a}\frac{\partial y}{\partial a} = 0$ so that $\left(\frac{\lambda_{AM}}{a}+\lambda_C\right)\left[\frac{\partial y}{\partial S}\frac{\partial S}{\partial a} + \frac{y}{a^2}\frac{\partial y}{\partial S}\frac{\partial S}{\partial(b\theta/a)}\right] + \frac{\lambda_{AM}}{a}\frac{\partial y}{\partial a} = 0.$

and the asset managers' asset holdings are

$$x_{AM}^{social} = 2\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_C\left(\Delta - \frac{\lambda_{AM}/a + \lambda_C}{\lambda_{AM} + \lambda_C}\psi + \Delta - \frac{\psi}{a}\right). \tag{33}$$

Equations (29)−(31) are the analogs of (19)−(21). As expected, the two sets of equations coincide when $\lambda_{AM} = 0$, and hence there is no externality. But so long as there are asset managers, the socially- and privately-optimal contracts are different. Our analysis below will reveal how exactly they compare to each other.

Notice how the equilibrium price in (32) compares to that in (22). The fact that the cost as perceived by the planner is higher than that perceived by an individual shareholder is reflected in the price, and reduces the price for a given a. This reflects the fact that the planner internalizes adverse effect of incentive provision and views it as being more costly compared to shareholders. This causes the planner to provide less incentives, which in particular means that $a^{social} < a^{private}$, as we will show in Proposition 2. Lower value of a further reduces the price, and thus $S^{social} < S^{private}$, as we will see in Proposition 3.

We are now ready to present the central result of the paper.

Proposition 2 (Socially- vs. Privately-Optimal Contracts). *(a) Compared to the equilibrium with the privately-optimal contract, the socially-optimal contract has a smaller absolute-performance sensitivity, that is,* $a^{social} < a^{private}$*;*

(b) Suppose that Assumption 1 holds. Then compared to the equilibrium with the privately-optimal contract, the socially-optimal contract involves less benchmarking, that is, $b^{social} < b^{private}$.[25]

As we have seen in our analysis, the use of incentive contracts inflates prices and thus *reduces the marginal benefit of incentive provision* for everyone else. The social planner internalizes this effect, and opts for less incentive provision than individual shareholders.

An interesting special case is the one when there are no conventional investors, i.e., $\lambda_C = 0$. Notice that in this case each asset manager will hold exactly $2\bar{x}$ and the total α in the economy is fixed, equal to $2\bar{x}^\top\Delta$. The planner understands that incentive provision is unnecessary in this case, and thus faces no tradeoff between incentive provision and incentives. Indeed, by substituting $\lambda_C = 0$ into (29)−(30), it immediately follows that the socially-optimal contract is $a = 1/2$ and $b = 0$, which provides perfect risk sharing and coincides with the first-best one (see Lemma 5 in the Appendix). Interestingly, the

[25] It is also true that $b^{social}/a^{social} < b^{private}/a^{private}$.

shareholders ignore the fact that, in equilibrium, their contracts will not help them generate higher returns, and use contracts with $a > 1/2$ and $b > 0$, as can be seen from (19)−(20).

To further emphasize that benchmarking is crucial for the comparison between privately- and socially-optimal contracts, consider a case where benchmarking is exogenously set to zero, $b = 0$. In this case, all incentive provision and risk sharing has to be done through a. As we discussed earlier, an increase in a has two opposing effects on the asset managers' demands and hence prices. As a result, it can be shown that with $b = 0$ the comparison between a^{social} and $a^{private}$ is ambiguous. Intuitively, both the marginal benefit of a (incentive provision) as well as its marginal cost (exposing the asset manager to more risk) are lower for the social planner who internalizes the effect of a on prices. Depending on which reduction is bigger, the planner can choose a higher or a lower a than the shareholders do. Thus, only because of benchmarking ($b \neq 0$) can we be sure of the direction of the externality and are able to say that privately-optimal contracts deliver excessive incentive provision.

We now show that excessive incentive provision and excessive benchmarking give rise to crowded trades and excessive costs.

Proposition 3 (**Crowded Trades and Excessive Costs of Asset Management**). *Compared to the equilibrium corresponding to the privately-optimal contract, in the equilibrium corresponding to the socially-optimal contract*

(a) asset prices are lower, $S^{social} < S^{private}$*;*

(b) asset managers' costs are lower, $\psi^\top x^{AM}_{social} < \psi^\top x^{AM}_{private}$*.*

As Proposition 3 shows, excessive use of incentive contracts by shareholders inflates prices and reduces per-share returns. In addition, the costs of asset management are excessively high. Our model thus contributes to the debate on whether costs of asset management are excessive and whether returns delivered by the managers justify these costs.

Finally, we discuss the benchmark weights. The same statement as in Lemma 3 applies to the case with socially-optimal contracts. In addition, we can compare the tilts to high value-added and/or high-cost stocks in the privately- and socially-optimal contracts.

Claim 1 (**Comparison of Benchmark Weights**). *Suppose that Assumption 1 holds. Then privately-optimal benchmark underweights stocks with higher* $\Delta - \psi$ *and overweighs stocks with higher* ψ *compared to the socially-optimal benchmark. Formally, consider two stocks* i *and* j*, that have the exact same characteristics except* $\Delta_i - \psi_i \geq \Delta_j - \psi_j$ *and* $\psi_i \leq \psi_j$*, with at least one inequality being strict. Then* $\theta^{social}_i - \theta^{social}_j > \theta^{private}_i - \theta^{private}_j$*.*

The intuition behind this result is a little tricky. Compare (21) and (31), and recall that the role of $b\theta$ is to protect the asset manager from risk as well as provide incentives, as captured by $(2a-1)$- and $(1-a)$-terms, respectively. The planner understands that the second role is less effective relative to how individual shareholders perceive it. That role is driven by ψ, and hence the planner is more reluctant to use benchmark weights to provide incentives for high-ψ stocks. And as the role of $b\theta$ in protecting the manager from risk is relatively more important than incentive provision, she will tilt the benchmark more towards high-value-added stocks than individual shareholders would.

Remark 1 (Redistribution Effects). Through our choice of weights in the social-welfare function, we have shut down the contracts' redistribution effects and isolated the pecuniary externality that the planner desires to correct. For certain applications (e.g., concerned with wealth inequality), however, it is useful to highlight the transfers from one set of agents to another that benchmarking generates. The answer is simple and it depends on agents' initial endowments or, more precisely, on whether an agent is a (net) buyer of stocks or a (net) seller. As we have argued, benchmarking boosts stock prices. This benefits (net) sellers of the stocks at the expense of (net) buyers. If the social planner favors investors who have high endowments of stocks and are planning to sell (e.g., the older generations), she has incentives to use more benchmarking in order to inflate prices so as to benefit them, and vice versa if she favors net buyers (who are typically the younger generations).

5 Conclusions

We consider the problem of optimal incentive provision for asset managers in a general-equilibrium asset-pricing model. The optimal contacts involve benchmarking. We show that by ignoring the effects of contracts on equilibrium prices, shareholders impose an externality on each other—the effectiveness of their incentive contracts is lower than they perceive it to be. The reason is that contracts incentivize the asset managers to invest more in stocks with higher α as well as stocks in the benchmark. This boosts prices and lowers returns, making the marginal benefit of α-chasing lower for everyone. The social planner, who internalizes the effects of contracts on equilibrium prices, opts for less incentive provision, less benchmarking, and lower asset-management costs.

As for directions for future research, it could be interesting to incorporate passive asset managers into the model. However, such an extension does present challenges. The existing evidence on the compensation contracts in the asset management industry covers only active funds. Very little is known about contracts of managers in passive funds. Before engaging in modeling of passive managers, it would be important to collect such evidence. A natural starting point would be to analyze the Statements of Additional Information filed by the U.S. mutual funds with the SEC, which contain information on managers' compensation structure. If contracts of passive managers turn out to be incentive contracts, it would be interesting to understand the incentive problem they solve. It is not at all obvious what kind of incentive problem would result in optimal contracts that make the (passive) managers closely follow the benchmark. We leave this problem for future work.

Appendix

Proof of Lemma 1. Equation (5) immediately follows from taking the first-order condition of problem (3) with respect to x. Similarly, (6) follows from taking the first-order condition of problem (4) with respect to x. $\square$

Lemma 5 (First Best). *If x is observable/contractible or if the private cost ψ is zero, then $a = 1/2$ and $b = 0$, and the asset prices are given by $S^{FB} = \mu - \gamma\Sigma\bar{x} + 2\lambda_{AM}(\Delta - \psi)$.*

Proof of Lemma 5. When x is contractible, the problem of the shareholder is simply to maximize $U^S + U^{AM}$. The first-best demand is

$$x_{AM} = \Sigma^{-1}\frac{\Delta - \psi + \mu - S}{\gamma[a^2 + (1-a)^2]} + (2a-1)\frac{b\theta}{a^2 + (1-a)^2}.$$

The first-order condition with respect to $b\theta$ is $\gamma\Sigma(y - z) = 0$. The first-order condition with respect to a is $\gamma\left[\Sigma(y-z)\right]^\top x + \gamma(1-2a)\sigma_\varepsilon^2 = 0$, which, using the first-order condition with respect to $b\theta$ immediately implies $a = 1/2$. Then setting $b = 0$ satisfies the first-order condition with respect to $b\theta$. The first-best demand is then

$$x_{AM} = \Sigma^{-1}\frac{\Delta - \psi + \mu - S}{\gamma/2}.$$

Finally, the expression for equilibrium asset prices in the first best is given by

$$S^{FB} = \mu - \gamma\Sigma\bar{x} + 2\lambda_{AM}(\Delta - \psi).$$

Compared to (22), $S^{FB} > S^{private}$.

Finally, notice that equilibrium holdings of the asset manager are

$$x_{AM}^{FB} = 2\left[\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_C(\Delta - \psi)\right].$$

Notice that if the asset manager holds positive amount of each stock in the first best, then part (a) of Assumption 1 must hold. Therefore part (a) of Assumption 1 is a necessary condition for no short-selling to occur in the first best. $\square$

Proof of Lemma 2. The first-order condition with respect to $b\theta/a$ is

$$
\begin{aligned}
0 &= \Delta - \psi + \mu - S - \gamma\Sigma z, \\
0 &= \Delta - \psi + \mu - S - \gamma\Sigma\left[\Sigma^{-1}\frac{\Delta - \psi/a + \mu - S}{\gamma}\left(\frac{1}{a} - 1\right) + \frac{b\theta}{a}\right], \\
\gamma\Sigma\frac{b\theta}{a} &= \Delta - \psi + \mu - S + \left(1 - \frac{1}{a}\right)\left(\Delta - \frac{\psi}{a} + \mu - S\right), \\
\gamma\Sigma\frac{b\theta}{a} &= \left(2 - \frac{1}{a}\right)(\Delta - \psi + \mu - S) + \left(1 - \frac{1}{a}\right)\left(1 - \frac{1}{a}\right)\psi.
\end{aligned}
$$

Using the expression for prices given in (7),

$$
\gamma\Sigma\frac{b\theta}{a} = \left(2 - \frac{1}{a}\right)\left[\Delta - \psi + \gamma\Sigma\Lambda\left(\bar{x} - \lambda_{AM}\frac{b\theta}{a}\right) - \frac{\lambda_{AM}}{a}\Lambda\left(\Delta - \frac{\psi}{a}\right)\right] + \left(1 - \frac{1}{a}\right)\left(\psi - \frac{\psi}{a}\right).
$$

Rearranging terms, and using (8),

$$
\begin{aligned}
&\gamma\Sigma\left[1 + \left(2 - \frac{1}{a}\right)\Lambda\lambda_{AM}\right]\frac{b\theta}{a} \\
&\quad = \gamma\Sigma\Lambda\left(2 - \frac{1}{a}\right)\bar{x} + \left(2 - \frac{1}{a}\right)\lambda_C\Lambda(\Delta - \psi) + \left[1 - \frac{1}{a} - \left(2 - \frac{1}{a}\right)\frac{\lambda_{AM}}{a}\Lambda\right]\left(\psi - \frac{\psi}{a}\right), \\
&\gamma\Sigma\Lambda\frac{b\theta}{a} = \Lambda\left(2 - \frac{1}{a}\right)\left[\gamma\Sigma\bar{x} + \lambda_C(\Delta - \psi)\right] - \left[\frac{\lambda_{AM}}{a} + \lambda_C\left(\frac{1}{a} - 1\right)\right]\Lambda\left(\psi - \frac{\psi}{a}\right), \qquad (34) \\
&b\theta = (2a - 1)\left[\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_C(\Delta - \psi)\right] + (1 - a)\left[\frac{1}{a} - \left(\frac{\lambda_{AM}}{a} + \lambda_C\right)\right]\frac{\Sigma^{-1}}{\gamma}\psi.
\end{aligned}
$$

Similarly, the first-order condition for b and θ evaluated at the equilibrium is given by (20) and (21), respectively.

Plugging (34) into the expression for prices (7), obtain

$$
\begin{aligned}
S^{private} &= \mu - \gamma\Sigma\Lambda\bar{x} + \lambda_{AM}\gamma\Sigma\Lambda b\theta + \Lambda\frac{\lambda_{AM}}{a}\left(\Delta - \frac{\psi}{a}\right) \\
&= \mu - \gamma\Sigma\Lambda\bar{x}\left(1 - \lambda_{AM}\left(2 - \frac{1}{a}\right)\right) + \lambda_{AM}(\Delta - \psi) + \frac{\lambda_{AM}\Lambda}{a}\left[\lambda_{AM} + a\lambda_C\right]\left(\Delta - \frac{\psi}{a}\right) \\
&= \mu - \gamma\Sigma\bar{x} + \lambda_{AM}\left(\Delta - \psi + \Delta - \frac{\psi}{a}\right).
\end{aligned}
$$

Substituting (16) into (6) and rearranging terms, obtain

$$\gamma \Sigma x^{AM} = (\Delta - \psi + \mu - S) + \left(\Delta - \frac{\psi}{a} + \mu - S\right).$$

Substituting the expression for prices $S = S^{private}$ derived above and rearranging terms yields (23). $\square$

Proof of Proposition 1. The result immediately follows from (20) and Assumption 1.$\square$

Proof of Lemma 3. Denote the (k, ℓ)-th element of matrix Σ^{-1} by $e_{k,\ell}$, where $e_{k,\ell} = e_{\ell,k}$ by symmetry, in particular, $e_{i,j} = e_{j,i}$. Since stocks i and j are assumed to be identical (except for Δ's and ψ's), we have $e_{i,i} = e_{j,j}$ and $e_{i,k} = e_{j,k}$ for all $k \neq i, j$ (i.e., stocks i and j have the same variance and covariance with other stocks). As a result,

$$\theta_i - \theta_j = (e_{i,i} - e_{i,j}) \left\{ \frac{2a-1}{b\gamma} \lambda_C [\Delta_i - \psi_i - (\Delta_j - \psi_j)] + \frac{1-a}{b\gamma} \left(\frac{1}{a} - \frac{\lambda_{AM}}{a} - \lambda_C \right) (\psi_i - \psi_j) \right\}.$$

Because Σ^{-1} is positive definite, we have $e_{i,i} > 0$, $e_{i,i}e_{j,j} - e_{i,j}^2 > 0$, $e_{i,i} > |e_{i,j}|$. As a result, $e_{i,i} - e_{i,j} > 0$, and thus $\theta_i > \theta_j$ whenever $\Delta_i - \psi_i \geq \Delta_j - \psi_j$, $\psi_i \geq \psi_j$, and at least one of the inequalities is strict. With a slight modification, this proof also applies to the socially-optimal contract. $\square$

The Social Planner's Problem in Terms of Exponential Utilities:

$$\max_{a,b,\theta,c,x=x^{AM},x^C} - \tilde{\omega}_S E \exp\left\{-\gamma \left[\left(x_{-1}^S\right)^\top S + r_x - (ar_x - br_{\mathbf{b}}) - c \right] \right\}$$
$$- \tilde{\omega}_C E \exp\left\{-\gamma \left[\left(x_{-1}^C\right)^\top S + \left(x^C\right)^\top (D - S) \right] \right\}$$

subject to (5), (6), and (9), where $\tilde{\omega}_i$, $i = S, C$, are the modified Pareto weights. From the first-order condition with respect to c it follows that the Lagrange multiplier on the participation constraint equals $\tilde{\omega}_S MU_S / MU_{AM}$, where MU_i denotes the expected marginal utility of agent i. This value is the effective Pareto weight on the asset manager's utility given that transfers between the shareholder and manager are allowed. Similarly, if transfers between shareholders and conventional investors were allowed, then $\tilde{\omega}_S MU_S = \tilde{\omega}_C MU_C$, and the redistribution effects would be zero. And without transfers, the Pareto weights that cancel out the redistribution effects (in the formulation with exponential utilities) are equal to inverse marginal utilities, $\tilde{\omega}_i = 1/MU_i$.

Proof of Lemma 4.

The social planner's first-order condition with respect to $b\theta/a$ is[26]

$$\left[\omega_S\left(x_{-1}^S - x^{AM}\right)^\top + \omega_C\left(x_{-1}^C - x^C\right)^\top\right]\frac{\partial S}{\partial(b\theta/a)}$$
$$+\,\omega_S\left[(\Delta-\psi+\mu-S-\gamma\Sigma z)^\top + (\Delta+\mu-S-\gamma\Sigma z)^\top\left(\frac{1}{a}-1\right)\frac{\partial y}{\partial S}\frac{\partial S}{\partial(b\theta/a)}\right] = 0.$$

Canceling out the redistribution effects and using $\partial y/\partial S = -\Sigma^{-1}/\gamma$ and $\partial S/\partial(b\theta/a) = \gamma\Sigma\Lambda\lambda_{AM}$, the above equation (or (25)) becomes

$$0 = \Delta-\psi+\mu-S-\gamma\Sigma z - (\Delta+\mu-S-\gamma\Sigma z)\frac{1-a}{a}\Lambda\lambda_{AM},$$
$$0 = \Delta-\psi+\mu-S-\gamma\Sigma z - \psi\frac{(1/a-1)\Lambda\lambda_{AM}}{1-(1/a-1)\Lambda\lambda_{AM}},$$
$$0 = \Delta-\psi+\mu-S-\gamma\Sigma\left[\Sigma^{-1}\frac{\Delta-\psi/a+\mu-S}{\gamma}\left(\frac{1}{a}-1\right)+\frac{b\theta}{a}\right] - \psi\frac{(1/a-1)\Lambda\lambda_{AM}}{1-(1/a-1)\Lambda\lambda_{AM}}.$$

Rearranging terms,

$$\gamma\Sigma\frac{b\theta}{a} = \Delta-\psi+\mu-S+\left(1-\frac{1}{a}\right)\left(\Delta-\frac{\psi}{a}+\mu-S\right) - \psi\frac{(1-a)/a\Lambda\lambda_{AM}}{1-(1/a-1)\Lambda\lambda_{AM}},$$
$$\gamma\Sigma b\theta = (2a-1)(\Delta-\psi+\mu-S) + (1-a)\left[\frac{1-a}{a} - \frac{\Lambda\lambda_{AM}}{1-(1/a-1)\Lambda\lambda_{AM}}\right]\psi,$$
$$\gamma\Sigma b\theta = (2a-1)(\Delta-\psi+\mu-S) + (1-a)\left(\frac{1}{a} - \frac{1}{\lambda_{AM}+\lambda_C}\right)\psi. \tag{35}$$

Alternatively,

$$0 = \Delta - \frac{\lambda_{AM}/a+\lambda_C}{\lambda_{AM}+\lambda_C}\psi + \mu - S - \gamma\Sigma z,$$
$$\gamma\Sigma\frac{b\theta}{a} = \Delta - \frac{\lambda_{AM}/a+\lambda_C}{\lambda_{AM}+\lambda_C}\psi + \mu - S + \left(1-\frac{1}{a}\right)\left(\Delta-\frac{\psi}{a}+\mu-S\right),$$
$$\gamma\Sigma b\theta = (2a-1)\left(\Delta - \frac{\lambda_{AM}/a+\lambda_C}{\lambda_{AM}+\lambda_C}\psi + \mu - S\right) + (1-a)\left(\frac{1}{a} - \frac{\lambda_{AM}/a+\lambda_C}{\lambda_{AM}+\lambda_C}\right)\psi.$$

[26]Because the asset manager's and conventional investor's utilities are maximized with respect to y and x^C, respectively, by the Envelope theorem the only terms from their payoffs that enter the first-order conditions are those entering the redistribution term.

Substituting the expression for prices into (35) leads to

$$b\theta = (2a-1)\left[\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_C(\Delta-\psi)\right] + (1-a)\left[\frac{1}{a} - \frac{\lambda_{AM}/a + \lambda_C}{\lambda_{AM}+\lambda_C}\right]\frac{\Sigma^{-1}}{\gamma}\psi,$$
$$b\theta = (2a-1)\left[\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_C(\Delta-\psi)\right] + (1-a)\left[\frac{1}{a} - 1\right]\frac{\lambda_C}{\lambda_{AM}+\lambda_C}\frac{\Sigma^{-1}}{\gamma}\psi.$$

Similarly, the first-order conditions for b and θ separately, evaluated in equilibrium, are then given by (30) and (31).

Following the same steps as in the proof of Lemma 2, the expression for prices is

$$S^{social} = \mu - \gamma\Sigma\bar{x} + \lambda_{AM}(\Delta-\psi) + \lambda_{AM}\left(\Delta - \frac{\psi}{a}\right) - \left(\frac{1}{a}-1\right)\frac{\lambda_{AM}^2}{\lambda_{AM}+\lambda_C}\psi,$$

or, alternatively,

$$S^{social} = \mu - \gamma\Sigma\bar{x} + \lambda_{AM}\left(\Delta - \frac{\lambda_{AM}/a + \lambda_C}{\lambda_{AM}+\lambda_C}\psi + \Delta - \frac{\psi}{a}\right).$$

Finally, substituting (27) into (6) and rearranging terms, obtain

$$\gamma\Sigma x^{AM} = \left[\Delta - \frac{\lambda_{AM}/a + \lambda_C}{\lambda_{AM}+\lambda_C}\psi + \mu - S\right] + \left[\Delta - \frac{\psi}{a} + \mu - S\right].$$

Substituting (32) and rearranging terms yields (33). □

Proof of Proposition 2.

(a) Comparison $a^{social} < a^{private}$ follows straightforwardly from comparing (19) and (29).

(b) Denote $a_1 = a^{private}$ and $a_2 = a^{social}$. From the first-order conditions with respect to b evaluated in equilibrium,

$$b_1 = (2a_1-1)\mathbf{1}^\top\left[\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_C(\Delta-\psi)\right] + (1-a_1)\left[\frac{1}{a_1} - \frac{\lambda_{AM}}{a_1} - \lambda_C\right]\mathbf{1}^\top\frac{\Sigma^{-1}}{\gamma}\psi,$$
$$b_2 = (2a_2-1)\mathbf{1}^\top\left[\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_C(\Delta-\psi)\right]$$
$$+ (1-a_2)\left[\frac{1}{a_2} - \frac{1}{\lambda_{AM}+\lambda_C}\left(\frac{\lambda_{AM}}{a_2} + \lambda_C\right)\right]\mathbf{1}^\top\frac{\Sigma^{-1}}{\gamma}\psi.$$

Under Assumption 1, in order to show that $b_1 > b_2$, it is sufficient to show that

$$(1-a_1)\left[\frac{1}{a_1} - \frac{\lambda_{AM}}{a_1} - \lambda_C\right] > (1-a_2)\left[\frac{1}{a_2} - \frac{1}{\lambda_{AM}+\lambda_C}\left(\frac{\lambda_{AM}}{a_2} + \lambda_C\right)\right],$$

which (given that both sides of the above inequality are positive) is equivalent to

$$\frac{(1-a_1)/a_1}{(1-a_2)/a_2}\frac{\lambda_{AM} + a_1\lambda_C + (1-2a_1)\lambda_C}{(\lambda_{AM}+a_2\lambda_C)\lambda_C/(\lambda_{AM}+\lambda_C) + (1-2a_2)\lambda_C} > 1. \tag{36}$$

Given the equilibrium conditions for a_1 and a_2, we have

$$\frac{1-a_1}{a_1^3(2a_1-1)} = \frac{1-a_2}{a_2^3(2a_2-1)}\frac{\lambda_C}{\lambda_{AM}+\lambda_C}. \tag{37}$$

Substitute this into inequality (36), obtain

$$\frac{a_1^2(2a_1-1)}{a_2^2(2a_2-1)}\frac{\lambda_C}{\lambda_{AM}+\lambda_C}\frac{\lambda_{AM} + a_1\lambda_C + (1-2a_1)\lambda_C}{(\lambda_{AM}+a_2\lambda_C)\lambda_C/(\lambda_{AM}+\lambda_C) + (1-2a_2)\lambda_C} > 1.$$

Since $a_1 > a_2$, it suffices to show that

$$\begin{aligned}
&\frac{\lambda_{AM} + a_1\lambda_C + (1-2a_1)\lambda_C}{(\lambda_{AM}+a_2\lambda_C)\lambda_C/(\lambda_{AM}+\lambda_C) + (1-2a_2)\lambda_C} > \frac{\lambda_C+\lambda_{AM}}{\lambda_C} \\
\Leftrightarrow\ & \lambda_{AM} + a_1\lambda_C + (1-2a_1)\lambda_C > \lambda_{AM} + a_2\lambda_C + (1-2a_2)(\lambda_C+\lambda_{AM}) \\
\Leftrightarrow\ & \lambda_{AM}(2a_2-1) > \lambda_C(a_1-a_2).
\end{aligned} \tag{38}$$

To show (38), use equation (37), rearranging which we get

$$\frac{1-a_1}{a_1^3(2a_1-1)}\frac{\lambda_{AM}}{\lambda_C} = \frac{1-a_2}{a_2^3(2a_2-1)} - \frac{1-a_1}{a_1^3(2a_1-1)},$$

or, equivalently,

$$\frac{\lambda_{AM}(2a_2-1)}{\lambda_C} = \frac{a_1^3}{1-a_1}\left[\frac{(1-a_2)(2a_1-1)}{a_2^3} - \frac{(1-a_1)(2a_2-1)}{a_1^3}\right].$$

The right-hand side of the above equation equals

$$\frac{-a_1^3 + 2a_1^4 - 2a_1^4 a_2 + a_2 a_1^3 - (-a_2^3 + 2a_2^4 - 2a_2^4 a_1 + a_1 a_2^3)}{(1-a_1)a_2^3}$$
$$= \frac{(a_1 - a_2)}{(1-a_1)a_2^3}[-(1+2a_1a_2)(a_1^2 + a_1a_2 + a_2^2) + 2(a_1+a_2)(a_1^2+a_2^2) + a_1a_2(a_1+a_2)].$$

Rearranging terms and doing some more algebra, yields

$$\frac{\lambda_{AM}(2a_2 - 1)}{\lambda_C(a_1 - a_2)}$$
$$= \frac{(2a_1-1)a_1^2(1-a_2) + (2a_2-1)a_2^2(1-a_1) + (2a_1-1)a_1a_2 + 2a_1a_2^2(1-a_1)}{a_2^3(1-a_1)}.$$

Since $1/2 < a_2 < a_1 < 1$,

$$\frac{\lambda_{AM}(2a_2 - 1)}{\lambda_C(a_1 - a_2)}$$
$$> \frac{(2a_1-1)a_1^2(1-a_2) + (2a_2-1)a_2^2(1-a_1) + (2a_1-1)a_1a_2 + a_2^3(1-a_1)}{a_2^3(1-a_1)} > 1.$$

Thus $b_1 > b_2$. □

Proof of Proposition 3.

(a) Follows immediately from comparing (22) and (32) and using part (a) of Proposition 2:

$$S^{private} - S^{social} = \lambda_{AM}\left(\frac{1}{a^{social}} - \frac{1}{a^{private}}\right)\psi + \left(\frac{1}{a^{social}} - 1\right)\frac{\lambda_{AM}^2}{\lambda_{AM} + \lambda_C}\psi,$$

both terms of which are strictly positive.

(b) Using (23) and (33),

$$\psi^\top\left(x_{social}^{AM} - x_{private}^{AM}\right) = \lambda_C \psi^\top \frac{\Sigma^{-1}}{\gamma}\psi\left[1 - \frac{\lambda_{AM}/a^{social} + \lambda_C}{\lambda_{AM} + \lambda_C} + \frac{1}{a^{social}} - \frac{1}{a^{private}}\right].$$

Since Σ^{-1} is positive definite and the expression in square brackets is negative (as $a^{social} < a^{private} < 1$), we have $\psi^\top\left(x_{social}^{AM} - x_{private}^{AM}\right) < 0$. □

Proof of Claim 1. Denote $a_1 = a^{private}$ and $a_2 = a^{social}$, and let $e_{i,j}$ be the (i,j)-th element of matrix Σ^{-1} as defined in the proof of Lemma 3. Then

$$\begin{aligned}
\theta_i^{private} - \theta_j^{private} &= \frac{2a_1 - 1}{b_1\gamma}\lambda_C(e_{i,i} - e_{i,j})[\Delta_i - \Delta_j - (\psi_i - \psi_j)] \\
&+ \frac{1-a_1}{b_1\gamma}\left(\frac{1}{a_1} - \frac{\lambda_{AM}}{a_1} - \lambda_C\right)(e_{i,i} - e_{i,j})(\psi_i - \psi_j), \\
\theta_i^{social} - \theta_j^{social} &= \frac{2a_2 - 1}{b_2\gamma}\lambda_C(e_{i,i} - e_{i,j})[\Delta_i - \Delta_j - (\psi_i - \psi_j)] \\
&+ \frac{1-a_2}{b_2\gamma}\left(\frac{1}{a_2} - \frac{\lambda_{AM}/a_2 + \lambda_C}{\lambda_{AM} + \lambda_C}\right)(e_{i,i} - e_{i,j})(\psi_i - \psi_j).
\end{aligned}$$

Using similar steps as in the proof of $b_1 > b_2$ in part (b) of Proposition 2 we can show that $b_1/(2a_1 - 1) > b_2/(2a_2 - 1)$ and thus $(2a_1 - 1)/b_1 < (2a_2 - 1)/b_2$. Furthermore,

$$\begin{aligned}
&\frac{1-a_1}{b_1}\left[\frac{1}{a_1} - \frac{\lambda_{AM}}{a_1} - \lambda_C\right] \\
&= \left[\mathbf{1}^\top \frac{\Sigma^{-1}}{\gamma}\psi + \frac{2a_1 - 1}{(1-a_1)(1/a_1 - \lambda_{AM}/a_1 - \lambda_C)}\mathbf{1}^\top\left(\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_C(\Delta - \psi)\right)\right]^{-1}, \\
&\frac{1-a_2}{b_2}\left[\frac{1}{a_2} - \frac{\lambda_{AM}/a_2 + \lambda_C}{\lambda_{AM} + \lambda_C}\right] \\
&= \left[\mathbf{1}^\top \frac{\Sigma^{-1}}{\gamma}\psi + \frac{2a_2 - 1}{(1-a_2)\left[1/a_2 - (\lambda_{AM}/a_2 + \lambda_C)/(\lambda_{AM} + \lambda_C)\right]}\mathbf{1}^\top\left(\bar{x} + \frac{\Sigma^{-1}}{\gamma}\lambda_C(\Delta - \psi)\right)\right]^{-1}.
\end{aligned}$$

From the proof of $b_1 > b_2$ in part (b) of Proposition 2 we know that

$$\frac{2a_1 - 1}{(1-a_1)(1/a_1 - \lambda_{AM}/a_1 - \lambda_C)} < \frac{2a_2 - 1}{(1-a_2)\left[1/a_2 - (\lambda_{AM}/a_2 + \lambda_C)/(\lambda_{AM} + \lambda_C)\right]},$$

and thus

$$\frac{1-a_1}{b_1}\left[\frac{1}{a_1} - \frac{\lambda_{AM}}{a_1} - \lambda_C\right] > \frac{1-a_2}{b_2}\left[\frac{1}{a_2} - \frac{\lambda_{AM}/a_2 + \lambda_C}{\lambda_{AM} + \lambda_C}\right].$$

Hence when $\Delta_i - \psi_i \geq \Delta_j - \psi_j$ and $\psi_i \leq \psi_j$, and at least one inequality is strict, we have $\theta_i^{social} - \theta_j^{social} > \theta_i^{private} - \theta_j^{private}$. And reverse, if $\Delta_i - \psi_i \leq \Delta_j - \psi_j$ and $\psi_i \geq \psi_j$, and at least one inequality is strict, then we have $\theta_i^{social} - \theta_j^{social} < \theta_i^{private} - \theta_j^{private}$. The interpretation is that the socially-optimal contract puts relatively less weight on incentive provision and thus relatively more weight on protecting the asset manager from risk. $\square$

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